$$\therefore z=\frac{H-h}{4}\left(\frac{H^2+2Hh+3h^2}{H^2+Hh+h^2}\right).$$

Let A, a be the lower and upper bases.

$$\therefore z=\tfrac{1}{4}(H-h)\left(\frac{A+2\sqrt{(Aa)}+3a}{A+\sqrt{(Aa)}+a}\right).$$

$H-h=20$ feet, $A=\frac{100}{144}=\frac{25}{36}$ square feet, $a=\frac{36}{144}=\frac{1}{4}$ square foot.

$\therefore z=8\frac{18}{49}$ feet. $20-8\frac{18}{49}=11\frac{31}{49}$.

Taking moments about the center of mass, $2x=11\frac{31}{49}$, $x=5\frac{40}{49}$, $8\frac{18}{49}-5\frac{40}{49}=2\frac{27}{49}$ feet from the larger end.

$2y=8\frac{18}{49}$, $y=4\frac{9}{49}$, $11\frac{31}{49}-4\frac{9}{49}=7\frac{22}{49}$ feet from the smaller end.

134. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

If $pv=Rt-b/tv$ be the equation for CO_2 gas, find the total, external and internal work done in compressing the gas from 102 to 136 atmospheres at a constant temperature 16°C, and constant volume, $R=63.23$, $b=481600$ for CO_2.

Solution by the PROPOSER.

136 atmospheres $=2000$ lbs. $=p_2$, 102 atmospheres $=1500$ lbs. $=p_1$.

$$pv=Rt-b/tv,\text{ or } v=\frac{Rt}{2p}+\frac{1}{2p}\sqrt{\frac{R^2t^3-4bp}{t}},\quad \frac{dv}{dt}=\frac{R}{2p}+\frac{R^2t^3+2bp}{2pt\sqrt{(R^2t^4-4bpt}}$$

$$=\frac{R}{2p}+\frac{b}{t\sqrt{(R^2t^4-4bpt)}}+\frac{R^2t^2}{2p\sqrt{(R^2t^4-4bpt)}}.$$

$$\text{External work}=t\int_{p_1}^{p_2}\left(\frac{dv}{dt}\right)dp$$

$$=t\int_{p_1}^{p_2}\left(\frac{R}{2p}+\frac{b}{t\sqrt{(R^2t^4-4bpt)}}+\frac{R^2t^2}{2p\sqrt{(R^2t^4-4bpt)}}\right)dp$$

$$=Rt\log_e\left(\frac{Rt^2-\sqrt{(R^2t^4-4bp_2t)}}{Rt^2-\sqrt{(R^2t^4-4bp_1t)}}\right)+\frac{1}{2t}[\sqrt{(R^2t^4-4bp_1t)}-\sqrt{(R^2t^4-4bp_2t)}].$$

Now $t=273+17=290$, $R=63.23$, $b=481600$.

$\therefore$ External work $=18336.7\log_e(1.3367)+46.087=5367.47$ ft. lbs.

$$\text{Total work}=\int_{p_1}^{p_2}vdp=\int_{p_1}^{p_2}\left(\frac{Rt}{2p}+\frac{1}{2p}\sqrt{\frac{R^2t^3-4bp}{t}}\right)dp$$

$$=Rt\log_e\left(\frac{Rt^2-\sqrt{(R^2t^4-4bp_2t)}}{Rt^2-\sqrt{(R^2t^4-4bp_1t)}}\right)+\frac{1}{t}[\sqrt{(R^2t^4-4bp_2t)}-\sqrt{(R^2t^4-4bp_1t)}].$$

$\therefore$ Total work=5275.3 ft. lbs.

$$\text{Internal work}=\int_{p_1}^{p_2}(v-dv/dt)dp=\frac{3}{2t}[\sqrt{(R^2t^4-4bp_2t)}-\sqrt{(R^2t^4-4bp_1t)}]$$

$$=-138.261 \text{ ft. lbs.}$$

Let γ be the ratio of specific heat at constant pressure to specific heat at constant volume; S=dynamic specific heat at constant volume; u=velocity of sound in CO_2, g=gravity, δ=density of mercury, d=density of CO_2, h=height of barometer.

Then $u=\left(\frac{g\delta h[1+(t/273)]\gamma}{d}\right)^{\frac{1}{2}}$. Let $t=0$, then $g=32.2$ feet. $\delta=13.59$, $h=29.92$ inches$=2.4935$ feet, $d=.00198$, u by experiment$=856$ feet.

$\therefore$ $\gamma=du^2/g\delta h=1.3296$.

At 0°C., $S=R/(\gamma-1)=63.23/.3296=191.84$.

For any temperature, t, $S_t=B+t\int_{\infty}^{v}(d^2p/dt^2)dv$.

$dp/dt=R/v+b/t^2v^2$, $d^2p/dt^2=-2b/t^3v^2$.

For $t=273$ absolute temperature and $v=1$ cubic foot, $S_t=191.84=B+2b/t^2v=B+963200/(273)^2$.

$\therefore$ $B=178.943$.

For $t=290°$ absolute or 17°C., $S_t=178.943+963200/(290)^2=190.396$.

$5273.3\div190.396=27.7$°C., the amount the temperature would rise if total work were converted to heat.

135. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, Ohio.

What force acting at an inclination ω with a horizontal line on the center of a wheel of given weight will roll the wheel over an immovable cylindric log whose diameter is $(1/m)$th that of the wheel?

Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, O.

If the meaning is to find the force F which will *start* the rolling the following is the solution:

If $\phi=\cos^{-1}\left(\frac{2\sqrt{m}}{m+1}\right)$=the angle the radius of the wheel through the point of contact with the log, and $\phi+\omega=\alpha$, the angle included by this radius and the direction of the required force, and W=the weight of the wheel, we have, taking moments about the point of contact,

$$F.r\sin\alpha=W.\frac{2r\sqrt{m}}{m+1}, \text{ giving } F.$$